Comprehensive Review of Assessments of Early Childhood Mathematics Competencies

Douglas H. Clements, Julie Sarama, Crystal Day-Hess, Carrie Germeroth, John Ganzar, Aleis Pugia, Jane Barker,

and the Development and Research in Early Mathematics Education (DREME) Network

***Note:* This is a brief form of a much more extensive report to be submitted for publication. Please contact Douglas.Clements@du.edu to be informed of the release of the complete report.**

The research reported here was supported by the Hiesing-Simons Foundation The opinions expressed are those of the authors and do not represent views of the Foundation.

Comprehensive Review of Assessments of Early Childhood Mathematics Competencies

There is presently a burgeoning interest in early mathematics. There are at least six reasons for recent interest in early mathematics. First, increasing numbers of children attend early care and education programs (Barnett, Hustedt, Hawkinson, & Robin, 2006; Doig, McCrae, & Rowe, 2003; Hinkle, 2000). Second, there is an increased recognition of the importance of mathematics (Doig, McCrae, & Rowe, 2003; National Research Council, 2001) and the varying performance of students in different countries in mathematics (Mullis et al., 1997), beginning in the preschool years (Blevins-Knabe & Musun-Miller, 1996; Ginsburg & Russell, 1981; Griffin, Case, & Capodilupo, 1995; Jordan, Huttenlocher, & Levine, 1992; Saxe, Guberman, & Gearhart, 1987; Starkey et al., 1999). Third, differences are not just between nations, but also between low- and higher-income children (Sarama & Clements, 2011; Thomson, Rowe, Underwood, & Peck, 2005) and different ethnic and cultural groups (NCES, 2000; Thomson et al., 2005). Fourth, early math competence predicts later math as well as later reading, suggesting that mathematics is a core component of cognition (Duncan et al., 2007; Duncan & Magnuson, 2011). Fifth, government agencies provide financial support for prekindergarten programs designed to facilitate academic achievement, particularly in low-income children, and research indicates that early interventions in mathematics can prevent later learning difficulties in school for all children (Clements & Sarama, 2007; Doig, McCrae, & Rowe, 2003; Fuson, Smith, & Lo Cicero, 1997; Griffin, 2004; Wright, 2003). Sixth, researchers and other educators have recognized that young children have greater potential to think and learn about mathematics than previously assumed (Clements & Sarama, 2014; Ginsburg, Lee, & Stevenson-Boyd, 2008; Sarama & Clements, 2009).

This attention to early mathematics generates a need for assessment instruments. However, there are only a few assessments of mathematics knowledge and skills for children as young as 3 to 4 years of age, with widely used extant instruments limited in scope of content. For example, a common instrument is the Applied Problems section of the Woodcock-Johnson IV. This subtest has several strengths, including assessing a wide range of abilities and ages, reliabilities above .80, and large normative data samples. However, two national panels on preschool assessment (NICHD Forum, Washington, DC, June 2002; CIRCL Forum, Temple University, January 30-31, 2003) cautioned sole use of the Woodcock-Johnson for assessment of mathematical skills in preschoolers because it has not been validated for children in the youngest age ranges; covers a narrow range of problems and jumps too quickly to advanced, formal knowledge; and is not based on current research of the development of mathematical thinking, including little attention to developmental sequences. Other assessments have similar limitations.

This situation has motivated multiple researchers and developers to create new instruments. These nascent efforts are not widely known and evaluation of the instruments is limited or unavailable. Such an analysis must be comprehensive, because different goals within the domains of policy, practice, and research require different types of assessments.

Perspectives

To ensure that the most appropriate mathematics assessment(s) are eventually used for research (and policy-oriented evaluations) and practice, we engaged in two related rounds of review, coding, and comparison. First, we identified and reviewed instruments designed to specifically measure mathematics competencies of children preschool through the primary grades, emphasizing those recently developed and frequently used in research, including their goals, foci, administration details, and psychometrics, and examine their appropriateness for various uses in early childhood education settings. Secondly, and distinct from most descriptions and reviews, our content analysis extended beyond simply listing topics covered by assessments, but also described how many items are included for each topic and what level of thinking they assess along research-based learning trajectories.

Methods and Data Sources

Mathematics-specific instruments focusing on the preschool through second grade age range were identified from comprehensive reviews of the literature and recommendations from a survey of experts in the field, resulting in extensive coding of 16 mathematics assessments. Data were collected on these assessments in a variety of categories, including: author(s), purpose/goals, grade/age, number of items, content areas covered, languages, time of administration, platform, administration format, training requirements, and psychometric information.

A particular focus of these analyses was the description and evaluation of the content assessed by each instrument. That is, not only did we count how many items each instrument had that measured each math topic, but we also (a) analyzed each major topic (e.g., number) into multiple subcomponents (subitizing, verbal counting, object counting, counting strategies, number comparison, etc.) and (b) analyzed the level of thinking along research-based learning trajectories (Sarama & Clements, 2009) for each item measuring each subcomponent. This extensive coding was conducted on 1976 assessment items across the assessments reviewed. This provides a far more comprehensive report of early childhood mathematics-specific assessments than has ever been available.

Two graduate research assistants were trained on all data entry and coding procedures (e.g., for assignment of items to topic and level of thinking within that topic). The first authors then checked all codes for the first two assessments; thereafter, any questions the GRA coders had were directed to the authors, and were checked via double-coding, with consensus reached for final coding.

Results

Findings and recommendations will be presented in various formats. First, the descriptive and comparative findings of each of the 16 assessments according to the established categories will be presented and discussed (Table 1). As part of this discussion, an internet link to a spreadsheet with thousands of cells will be provided that further describe the content covered in each assessment (the website component is currently under development, but a sample of information to be provided is presented in Tables 2 and 3).

For the purposes of this brief overview, in the remainder of this section, we provide an example of the text on “content and levels of thinking measured” for selected instruments—recognizing that additional information (presented in Table 1) will also be reviewed in the forthcoming publication, including the spreadsheets presented in Tables 2 and 3 containing complementary information on the topic and level of thinking (from research-based learning trajectories) of every item on each instrument.

An in-depth item-level analysis of nearly 2000 items on 14 of 16 assessments identified for inclusion in this paper was conducted to determine the level of thinking assessed along research-based learning trajectories (LTs). Twenty-one LT topics identified as key to mathematical thinking and development (Sarama & Clements, 2009, see Table 2 for comprehensive list of all LTs) were included in coding. Also included were two topics not considered key learning trajectories, but which occurred with enough frequency to warrant inclusion as separate coded content areas (Numeral Naming/Writing and Other STEM Measurement [e.g., telling time]). One LT Topic is reviewed here; data are presented for the 20 LT and two additional topics in the full article. Half of the assessments reviewed at this level focused only on number-related skills, while the remaining seven assessments included items on number-related skills *and* geometry, algebraic thinking, and/or geometric measurement skills (Table 2). Adding/Subtracting and Counting were the most frequently represented number topics, while Subitizing, Composing Numbers, and Fractions were represented far less frequently. Shapes was the most frequently represented non-number topic, while some topics were not represented in any of the assessments reviewed (i.e., Disembedding Geometric Figures, Composing 3D Shapes, and Volume Measurement).

The REMA Full (Clements, Sarama, & Liu, 2008) and TEAM (Clements, Sarama, & Liu, 2008; Clements, Sarama, & Wolfe, 2011/2016) include items that represent 17 of 21 LTs (81%) – seven more LTs than the next closest measure (excluding the related REMA-Short Form measure, which covers six of the LTs). The REMA and TEAM were designed to measure the math proficiencies of conceptual understanding, fluency, strategic competence, and reasoning of children three to eight years of age. Scoring of the REMA and TEAM assessments is unique in that they include codes to record the level of thinking and strategies children use in attempting to solve most assessment items – children's levels of thinking and item difficulty are then included to determine IRT T-scores. Learning trajectories in number include numeral, subitizing, counting, comparing number, adding/subtracting, composing number, multiplying/dividing, and fractions. These domains help distinguish between children who have not constructed true number concepts and those who have. Geometry LTs represented include spatial visualization, shape, composing shapes, 3D shapes, length measurement, area measurement, angle measurement, patterning, and classifying and analyzing data. These measures also include "Other STEM Measurement" items (e.g., telling time). Table 3 shows that, by purposeful design, these assessments cover many levels of the learning trajectories.

Though limited in its age range (appropriate for preschool-aged children), the Every Child Ready – Math (ECR-M AppleTree Institute for Education Innovation, 2010) includes the next highest percentage of LTs (48%). The ECR-M is designed to provide clear and actionable information to teachers and families on areas of strength and challenge in mathematical development of preschool-aged children. Ten of 21 LT topics are covered in the assessment, including: Numeral, subitizing, counting, comparing number, adding/subtracting, spatial orientation, shape, length measurement, patterning, and classifying and analyzing data. The assessment also includes items assessing Numeral Naming & Writing and Other STEM Measurement – not key mathematics LT topics.

The Early Math Diagnostic Assessment (EMDA, Pearson, 2003) which covers 43% of LTs, is designed to guide and inform instruction and evaluates the mathematics skills of young learners in grades PreK through third grade. Nine of 21 LT topics are covered in the assessment, including: Counting, comparing number, adding/subtracting, composing numbers, fractions, shape, length measurement, patterning, and classifying and analyzing data. The assessment also includes items assessing "Numeral Naming & Writing and Other STEM Measurement."

The REMA-Brief (Clements, Sarama, & Liu, 2008) is a condensed version of the previously described REMA and TEAM assessments, including 20 assessment items focused on the assessment of pre-kindergarten children. The REMA-Brief covers eight of the 21 LTs (38%): Numeral, subitizing, counting, comparing numbers, composing number, shape, composing shapes, and patterning.

The remaining seven assessments reviewed only cover number-related LTs. Assessments include: IGDI-EN (Hojnoski & Floyd, 2013), mCLASS-Math (Amplify Education, 2016), PENS-B (Purpura & Lonigan, 2015), PENS (Purpura & Lonigan, 2015) TEMA-3 (Ginsburg & Baroody, 2003), WJ-IV Applied Problems, and WJ-IV Calculations (Schrank, McGrew, Mather, & Woodcock, 2014) Though this more narrow focus may be desirable depending on the goals of mathematics assessment, it is important for researchers and practitioners to be aware of the content covered, as well as that not covered. These assessments varied widely in terms of how many number LTs covered, ranging from one to six of eight LT topics (Table 2), with an even wider range of levels of thinking covered (e.g., Table 3).

Significance for Science, Policy, and Practice

The increased attention to promoting high-quality mathematics education has elevated the demands for valid and useful measures of mathematical thinking and learning. Extending our knowledge of young children's mathematical development and evaluating the effectiveness of programs designed for them require accurate measures of their mathematical knowledge and skill. To date, however, no comprehensive reviews of early childhood math-specific assessments exists to aide researchers and practitioners in choosing the most appropriate assessments to meet their needs. To address this gap, we identified, described, and analyzed recently developed and oft-used instruments for mathematics assessment.

Our findings indicate that there are instruments that meet criteria for different purposes, but they different substantially in their logistical and content-coverage characteristics, so they should be considered and compared carefully before being selected to meet specific goals within the domains of policy, practice, and research. More specifically, we found that most widely-used instruments are useful for certain purposes, but are limited in terms of the content areas covered and the ability to capture children's level of thinking. These findings, and the detailed content coding made available in this work, provide a far more comprehensive tool for the selection of instruments than has previously been available.

References

Amplify Education. (2016). *mCLASS Matrh*. Brooklyn, NY: Author.

AppleTree Institute for Education Innovation. (2010). *Every child ready: Math assessment*. Washington, DC: Author.

Barnett, W. S., Hustedt, J. T., Hawkinson, L. E., & Robin, K. B. (2006). *The state of preschool 2006*. New Brunswick, NJ: National Institute for for Early Education Research (NIEER).

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5*, 35-45. doi: 10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the *Building Blocks* project. *Journal for Research in Mathematics Education, 38*(2), 136-163.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). New York, NY: Routledge.

Clements, D. H., Sarama, J., & Liu, X. (2008). Development of a measure of early mathematics achievement using the Rasch model: The Research-based Early Maths Assessment. *Educational Psychology, 28*(4), 457-482. doi: 10.1080/01443410701777272

Clements, D. H., Sarama, J., & Wolfe, C. B. (2011/2016). *TEAM—Tools for early assessment in mathematics*. Denver, CO: Kennedy Institute, University of Denver.

Doig, B., McCrae, B., & Rowe, K. (2003). A good start to numeracy: Effective numeracy strategies from research and practice in early childhood. Retrieved from Australian Council for Educational Research website: http://www.dest.gov.au/NR/rdonlyres/D2B70D11-FBE8-45CA-8D27-30408C8BF7DF/3986/GoodStart.pdf

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428–1446. doi: 10.1037/0012-1649.43.6.1428

Duncan, G. J., & Magnuson, K. (2011). The nature and impact of early achievement skills, attention skills, and behavior problems. In G. J. Duncan & R. Murnane (Eds.), *Whither opportunity? Rising inequality and the uncertain life chances of low-income children* (pp. 47-70). New York, NY: Russell Sage.

Fuson, K. C., Smith, S. T., & Lo Cicero, A. M. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*, 738-760.

Ginsburg, H. P., & Baroody, A. J. (2003). *Test of Early Mathematics Ability–Third edition* (3rd ed.). Austin, TX: Pro-ed. doi: 10.1177/0734282905282839

Ginsburg, H. P., Lee, J. S., & Stevenson-Boyd, J. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22*(1), 1-24.

Ginsburg, H. P., & Russell, R. L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development, 46*(6, Serial No. 193).

Griffin, S. (2004). Number Worlds: A research-based mathematics program for young children. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 325-342). Mahwah, NJ: Erlbaum.

Griffin, S., Case, R., & Capodilupo, A. (1995). Teaching for understanding: The importance of the Central Conceptual Structures in the elementary mathematics curriculum. In A. McKeough, J. Lupart & A. Marini (Eds.), *Teaching for transfer: Fostering generalization in learning* (pp. 121-151). Mahwah, NJ: Erlbaum.

Hinkle, D. (2000). *School involvement in early childhood*. Washington, DC: National Institute on Early Childhood Development and Education, U.S. Department of Education Office of Educational Research and Improvement.

Hojnoski, R. L., & Floyd, R. G. (2013). *Individual growth and development indicators of early numeracy (IGDIs-EN)*. Minneapolis, MN: Early Learning Labs.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology, 28*, 644-653.

Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1997). *Mathematics achievement in the primary school years: IEA's third international mathematics and science study (TIMSS)*. Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

NCES. (2000). *America's kindergartners (NCES 2000070)*. Washington, DC: National Center for Education Statistics, U.S. Government Printing Office.

Pearson. (2003). *Early math diagnostic assessment*. San Antonio, TX: Author.

Purpura, D. J., & Lonigan, C. J. (2015). Early Numeracy Assessment: The Development of the Preschool Numeracy Scales. *Early Education and Development, 26*(2), 286-313. doi: 10.1080/10409289.2015.991084

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.

Sarama, J., & Clements, D. H. (2011). Mathematics knowledge of low-income entering preschoolers. *Far East Journal of Mathematical Education, 6*(1), 41-63.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52*(2, Serial #216).

Schrank, F. A., McGrew, K. S., Mather, N., & Woodcock, R. W. (2014). *Woodcock-Johnson IV Tests of Cognitive Abilities*. Rolling Meadows, IL: Riverside.

Starkey, P., Klein, A., Chang, I., Qi, D., Lijuan, P., & Yang, Z. (1999, April). *Environmental supports for young children's mathematical development in China and the United States.* Paper presented at the Society for Research in Child Development, Albuquerque, NM.

Thomson, S., Rowe, K., Underwood, C., & Peck, R. (2005). Numeracy in the early years: Project Good Start. Retrieved from Australian Council for Educational Research website: http://www.acer.edu.au/documents/GOODSTART_FinalReport.pdf

Wright, R. J. (2003). A mathematics recovery: Program of intervention in early number learning. *Australian Journal of Learning Disabilities, 8*(4), 6-11.

Table 1. Summary of Early Childhood Mathematics Assessments Reviewed

Assessment	Ages	Content & Levels of Thinking Measured				Languages, Administration, Time, & Training				Demonstrated Applicability to ECE		
Name	Grade/ Age	Number of Items	Content Areas Covered (Learning Trajectories)	Scoring Goes Beyond Correctness*	Goes Beyond Number	Languages	Administration Time	Training Required	Certification Required to Administer	Screening	Formative Assessment	Serving Summative Research
Birthday Party Assessment (Ginsberg & Pappas, 2016)	3-5 years	8-11 tasks (30-36 items total) depending on age of child	Counting, Comparing Number, Adding/Subtracting, Spatial Orientation, Shape, Area Measurement, Patterning (Numeral Naming/Writing)	*	✓	English, Spanish	20 min.	Training for formative assessment provided via three workshops and website		✓	✓	✓
Child Math Assessment (Starkey & Klein, 2004)	3-5 years	16 tasks (multiple items per task)	Not Obtained for Item-Level Coding	✓	✓	English, Spanish	2 sessions, 20-30 min./ea.	Not reported				✓
Early Math Diagnostic Assessment (Pearson Inc., 2003)	PK-3	64	Counting, Comparing Number, Adding/Subtracting, Composing Numbers, Fractions, Shape, Length Measurement, Patterning, Classifying & Analyzing Data, Other STEM Measurement (Numeral name/writing)	*	✓	English	15-20 min.	No formal training; assessment instructions provided in test package			✓	✓
ECLS-K (NCES, 1998a)	K	93	Not Obtained for Item-Level Coding		✓	English, Spanish	Depends on student progress	Training specific to ECLS study; approx. 2-day administration training	✓			✓
ECLS-K: 2011 (NCES 2015)	K	17 routing items (1st stage); performance determines number of items in 2nd stage	Not Obtained for Item-Level Coding		✓	English, Spanish	Depends on student progress	Training specific to ECLS study; approx. 6-day administration training. Also included home study piece	✓			✓
ECLS-B (NCES 2009)	PK-K	PK: 28 item core; 9 item basal and ceiling supplement; K: 42 items	Not Obtained for Item-Level Coding		✓	English, Spanish	Depends on student progress	Training specific to ECLS study; approx. 2-day administration training	✓			✓

Assessment	Ages	Content & Levels of Thinking Measured				Languages, Administration, Time, & Training				Demonstrated Applicability to ECE		
Name	Grade/ Age	Number of Items	Content Areas Covered (Learning Trajectories)	Scoring Goes Beyond Correctness*	Goes Beyond Number	Languages	Administration Time	Training Required	Certification Required to Administer	Screening	Formative Assessment	Serving Summative Research
Every Child Ready - Math (AppleTree Institute for Education Innovation, 2010)	3-4 years	33	Numeral, Subitizing, Counting, Comparing Number, Adding/Subtracting, Spatial Orientation, Shape, Length Measurement, Patterning, Classifying & Analyzing Data, Other STEM Measurement (Numeral Naming/Writing)		✓	English	15-20 min.	Standardized 45-min training session; at least one in-person fidelity obs. and score; one video-recorded assessment	✓		✓	✓
Individual Growth & Development Indicators -- Early Numeracy (IGDI-EN) (Hojnoski & Floyd, 2013)	3-6 years	4 tasks (multiple items per task)	Counting, Comparing Number, (Numeral Naming/Writing)			English	5-10 min.	Training session, typically followed by 1:1 observation		✓	✓	
mCLASS: Math (mCM) (Amplify Education, 2016)	K-3	Varies; all tasks are timed, proceed until reach one minute	Numeral, Counting, Comparing Number, Adding/Subtracting, Multiplying/Dividing, Fractions, (Numeral Naming/Writing)	*		English, Spanish	10-12 min.	One-day onsite session options available on general administration		✓	✓	✓
Number Sense (Schacter, 2014)	PK	23-62, depending on version	Not Obtained for Item-Level Coding			English	5-10 min.	None; assessment interface provides instructions				✓
PENS (Purpura & Lonigan, 2015)	PK-K	68	Numeral, Subitizing, Counting, Comparing Number, Adding/Subtracting, Multiplying/ Dividing, (Numeral Naming/Writing)			English	3-4 sessions, 20 min./ea	Developed for classroom use; training not currently established		✓	✓	✓
PENS-B (Purpura, Reid, Elland, & Baroody, 2015)	PK-K	24	Numeral, Counting, Comparing Number, Adding/Subtracting			English	5 min.	Developed for classroom use; training not currently established		✓	✓	✓
REMA—Brief (Clements, Sarama, & Liu, 2008)	PK-K	20	Numeral, Subitizing, Counting, Comparing Number, Composing Numbers, Shape, Composing Shapes, Patterning		✓	English, Spanish	10-15 min.	Training session, with individual practice sessions (taped w/ feedback from developers)	✓	✓	✓	✓

Assessment	Ages	Content & Levels of Thinking Measured				Languages, Administration, Time, & Training				Demonstrated Applicability to ECE		
Name	Grade/ Age	Number of Items	Content Areas Covered (Learning Trajectories)	Scoring Goes Beyond Correctness*	Goes Beyond Number	Languages	Administration Time	Training Required	Certification Required to Administer	Screening	Formative Assessment	Serving Summative Research
REMA Full (Clements, Sarama, & Liu, 2008)	PK-2	197	Numeral, Subitizing, Counting, Comparing Number, Adding/Subtracting, Composing Numbers, Multiplying/Dividing, Fractions, Spatial Visualization, Shape, Composing Shapes, 3D Shapes, Length Measurement, Area Measurement, Angle Measurement, Patterning, Classifying & Analyzing Data, Other STEM Measurement	✓	✓	English, Spanish	2 sessions, 30-45 min./ea.	Training session, with individual practice sessions (taped w/ feedback from developers)	✓	✓	✓	✓
REMA—Short Form (Clements, Sarama, & Liu, 2008)	PK-2	80	Numeral, Subitizing, Counting, Comparing Number, Adding/ Subtracting, Composing Numbers, Multiplying/Dividing, Fractions, Shape, Composing Shapes, 3D Shapes, Length Measurement, Area Measurement, Angle Measurement, Patterning, Classifying & Analyzing Data	✓	✓	English, Spanish	10-15 min.	Training session, with individual practice sessions (taped w/ feedback from developers)	✓	✓	✓	✓
TEAM (Clements & Sarama, 2011; Clements, Sarama, & Liu, 2008)	PK-2	157	Numeral, Subitizing, Counting, Comparing Number, Adding/Subtracting, Composing Numbers, Multiplying/ Dividing, Fractions, Spatial Visualization, Shape, Composing Shapes, 3D Shapes, Length Measurement, Area Measurement, Angle Measurement, Patterning, Classifying & Analyzing Data, Other STEM Measurement	✓	✓	English, Spanish	2 sessions, 30-45 min./ea.	Training session, with individual practice sessions (taped w/ feedback from developers)	✓	✓	✓	✓
TEMA-3 (Ginsberg & Baroody, 2003)	3-8 years	72	Numeral, Subitizing, Counting, Adding/Subtracting, Composing Numbers, Multiplying/Dividing, (Numeral Naming/Writing)	✓		English	45 min.	No formal training provided; assessment instructions in test package		✓	✓	✓
WJ IV - Applied Problems (Schrank, McGrew, & Mather, 2014)	PK-adult	20 (until 4th grade starting point)	Numeral, Counting, Comparing Number, Adding/Subtracting, Multiplying/Dividing, Other STEM Measurement			English	10 min.	Included basic training covers administration, scoring, and interpretation guidelines				✓
WJ IV - Calculation (Shrank, McGrew, & Mather, 2014)	PK-adult	16 (until 4th grade starting point)	Adding/Subtracting, Multiplying/Dividing			English	10 min.	Included basic training covers administration, scoring, and interpretation guidelines				✓

*Scores include correctness only, but assessors have the option of recording details of children's errors and mathematical thinking.

Table 2. LT Topic Counts by Assessment

Learning Trajectory Topic	Birthday Party Assessment	EMDA	ECR-M	IGDI-EN	mCLASS: Math	PENS-B	PENS	REMA Brief	REMA Full	REMA Short Form	TEAM	TEMA-3	WJ IV - Applied Problems	WJ IV - Calculations	Total Per Learning Trajectory
Numeral	0	0	8	0	35	4	14	1	1	1	1	2	1	0	**68**
Numeral Naming/ Writing (non-LT)	3	4	5	66	70	0	4	0	0	0	0	10	0	0	**162**
Subitizing	0	0	2	0	0	0	7	4	6	3	4	1	0	0	**27**
Counting	20	9	18	2	179	7	24	6	58	22	48	30	10	0	**433**
Comparing Number	10	6	8	32	98	7	10	2	23	7	18	4	1	0	**226**
Adding/Subtracting	11	19	8	0	208	7	17	0	96	51	72	21	11	20	**541**
Composing Numbers	0	3	0	0	0	0	0	2	12	8	11	0	0	0	**36**
Multiplying/Dividing	0	0	0	0	23	0	11	0	41	18	41	2	1	6	**143**
Fractions	0	11	0	0	12	0	0	0	5	3	5	0	0	0	**36**
Spatial Orientation	12	0	3	0	0	0	0	0	0	0	0	0	0	0	**15**
Spatial Visualization	0	0	0	0	0	0	0	0	2	0	2	0	0	0	**4**
Shape	21	1	10	0	0	0	0	6	26	6	24	0	0	0	**94**
Composing Shapes	0	0	0	0	0	0	0	1	8	5	6	0	0	0	**20**
Disembedding Geometric Figures (2D)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
Composing Shapes (3D)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
3D Shapes	0	0	0	0	0	0	0	0	2	1	2	0	0	0	**5**
Length Measurement	0	3	10	0	0	0	0	0	15	6	11	0	0	0	**45**
Area Measurement	2	0	0	0	0	0	0	0	6	4	6	0	0	0	**18**
Volume Measurement	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
Angle Measurement	0	0	0	0	0	0	0	0	8	2	5	0	0	0	**15**
Patterning	7	2	4	0	0	0	0	1	20	10	13	0	0	0	**57**
Classifying & Analyzing Data	0	4	7	0	0	0	0	0	1	1	1	0	0	0	**14**
Other STEM Measurement (non-LT)	0	2	8	0	0	0	0	0	3	0	3	0	1	0	**17**
Total Items**	**86**	**64**	**91**	**100**	**625**	**25**	**87**	**23**	**333**	**148**	**273**	**70**	**25**	**26**	**1976**

*Child Math Assessment, ECLS, and Number Sense are not included here because full copies of the assessments could not be obtained for item-level review.
**Some items on the REMA Brief, REMA Full, REMA Short Form, and TEAM were coded for multiple LT levels due to the scoring procedures used - item scoring includes information about child errors and strategy use, allowing for multiple levels to be scored.

Table 3. Counting LT Counts by Assessment

		Assessment													
LT Levels	**LT Level Definitions**	**Birthday Party**	**EMDA**	**ECR-M**	**IGDI -EN**	**mCLASS: Math**	**PENS**	**PENS-B**	**REMA Brief**	**REMA Full**	**REMA Short Form**	**TEAM**	**TEMA-3**	**WJ IV-App. Prob.**	**Wj IV - Calc.**
Pre-Counter	No verbal counting														
Chanter	Verbal Chants “sing-song” or sometimes indistinguishable number words.														
Reciter	Verbally counts with separate words, not necessarily in the correct order above “five”.								1	1	1	1			
Reciter (10)	Verbally counts to ten, with some correspondence with objects, but may either continue an overly rigid correspondence, or exhibit performance errors (e.g., skipping, double-counting)	4		1	1	1		1	1	1	1	1	1		
Corresponder	Keeps 1-1 correspondence between counting words and objects, at least for small groups of objects laid in a line. May answer a “how many?” question by re-counting the objects, or violate 1–1 or word order.			1			5	3	1	4	2	2	2		
Counter (Small Numbers)	Accurately counts objects in a line to 5 and answers the “how many” question with the last number counted. When objects are visible, and especially with small numbers, begins to understand cardinality.		5				2	1		1		3	2	8	
Counter (10)	Counts arrangements of objects to 10. May be able to write numerals to represent 1–10. May be able to tell the number just after or just before another number, but only by counting up from 1.	3	1	1			2		1	5	1	3	3		
Producer (Small Numbers)	Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.			1			2			1	1		4	2	
Counter and Producer (10+)	Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality. Keeps track of objects that have and have not been counted, even in different arrangements. Writes or draws to represent 1 to 10 (then, 20, then 30).	4		1	1		7	1		9	2	7	1		
Counter Backward from 10	Counts backward from 10 to 1, verbally, or when removing objects from a group.									1		1	1		
Counter from N (N+1, N-1)	Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the number of counts).	2	1	1		65	6	1	1	3	1	2	1		
Skip Counter by 10s to 100	Skip counts by tens up to 100 or beyond with understanding; e.g., “sees” groups of 10 within a quantity and count those groups by 10.			3		11				1	1	1	2		
Counter to 100	Verbal Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number.					69			1	5	2	4	4		
Counter On Using Patterns	Keeps track of a few counting acts, but only by using numerical pattern (spatial, auditory, or rhythmic).		1							4	2	4			
Skip Counter	Counts by fives and twos with understanding.		1	3		33				3	1	3	1		
Counter of Imagined Items	Counts mental images of hidden objects.	7		2											
Counter On Keeping Track	Keeps track of counting acts numerically, first with objects, then by “counting counts.” Counts up 1 to 4 more from a given number.									9	4	7			
Counter of Quantitative Units/Place Value	Understands the base-ten numeration system and place-value concepts, including ideas of counting in units and multiples of hundreds, tens, and ones. When counting groups of 10, can decompose into 10 ones if useful.									3	2	3	5		
Counter to 200	Counts accurately to 200 and beyond, recognizing the patterns of ones, tens, and hundreds.									2		2	1		
Number Conserver	Consistently conserves number even in face of perceptual distractions such as spreading out objects of a collection.			4											
Counter Forward and Back	Counts “counting words” (single sequence or skip counts) in either direction. Recognizes that decades sequence mirrors single-digit sequence.									5	1	4			
Counter to 1000	Counts accurately to 1000 and beyond, recognizing the patterns of ones, tens, hundreds, and thousands.												2		